# Playing Board Games to Learn Rational Numbers: A Proof-of-Concept

Jairo A. Navarrete-Ulloa[1] and Felipe Munoz-Rubke[2]

**ABSTRACT**— Educational board games are a promising teaching method due to their low cost, playful, exploratory, and engaging nature. By drawing on analogical research, we created a game whose structure of spatial relationships mirrored the structure of rational numbers. We expected that children playing this game would improve their knowledge of fractions. We conducted a school intervention with an active control group and pretest-posttest assessments to evaluate our board game. Playing this game promoted the learning of fractions, even after controlling for nonverbal cognitive abilities. This low-cost educational game might help reduce the knowledge gap that separates less and more affluent children.

Many students struggle with fractions. The need to improve its teaching has led to the creation of novel tools such as targeted curricula (Saxe et al., 2007) and digital learning games (Kiili, Moeller, & Ninaus, 2018). However, board games have yet to be considered as a vehicle to promote the learning of fractions. Board games can promote creativity, concentration, engagement, collaboration, and confidence, and are an effective tool for teaching early mathematics (Ramani et al., 2019; Ramani, Siegler, & Hitti, 2012; Siegler & Ramani, 2008, 2009). They adapt well to outside-of-school contexts and fit the preferences of children who expect learning activities to be fast, active, and exploratory (Kirriemuir & McFarlane, 2004). Their affordability and low prior knowledge requirements make them educational tools accessible to rural areas and places with socioeconomic vulnerability.

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The design of educational games (EDGs) requires a framework guiding the mapping between game dynamics and the development of new skills (for a review, see Kiili, Koskinen, & Ninaus, 2019). Prior research has shown that educational analogies embedded into spatial domains such as board games help scaffold basic and intuitive learning of natural numbers (Navarrete, Gómez, & Dartnell, 2018; Ramani et al., 2012; Ramani et al., 2019; Siegler & Ramani, 2008, 2009). In this study, we explore if this method can be extended to the learning of fractions.

The following section describes the theoretical background that guided the design of an EDG for the teaching of fractions on the basis that well-designed educational analogies foster reasoning, learning, and transfer (Gentner, Loewenstein, & Thompson, 2003; Holland, Holyoak, Nisbett, & Thagard, 1989; Mayer & Alexander, 2016; Richland, Zur, & Holyoak, 2007).

### Educational Analogies and Our Board Game

The "Building Fractions" game aims to help children master critical conceptual knowledge about fractions. That is, help them to develop abilities to associate fractions with their location on a number line (i.e., a measure), think of the addition of two fractions as added magnitudes, apply concepts of equivalence/comparison, and operate with the part-whole relation (Charalambous & Pitta-Pantazi, 2006; Gabriel et al., 2013; Hamdan & Gunderson, 2017). As analogical reasoning helps children understand phenomena as systems of structured relationships that can be aligned, compared, and mapped together (Richland & Simms, 2015), we designed our game to provide a system of spatial relationships that mirrors the critical structure of rational numbers.

We grounded our work in a model describing how students learn fractions by building an analogical relationship between the structure of rational numbers and a concrete domain with a compatible structure (Navarrete & Dartnell, 2017). The analogical relationship comprises spatial-numerical mappings between relational concepts

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**Table 1**
The Analogical Relationship Maps Relational Concepts (H-Mappings) and Concrete Objects (h-Mappings)

| *Spatial domain* | | *Numeric domain* | *Level* |
|---|---|---|---|
| Stacking of blocks | → | Adding fractions | Relational (H) |
| Unstacking blocks | → | Subtracting fractions | Relational (H) |
| Length equivalence of blocks | → | Numerical equivalence of fractions | Relational (H) |
| Length comparison of blocks | → | Numerical comparison of fractions | Relational (H) |
| Comparing blocks against a block representing a single unit | → | Estimation on a number line | Relational (H) |
| Block with length L | → | Fraction symbol for length L | Objects (h) |

**Table 2**
Actions on the Spatial Configuration of Blocks Can Be Translated into Numerical Statements and Operations

| *Spatial actions and perceptions* | *Learning contents* |
|---|---|
| Stacking blocks X and Y generates block Z | $h(X) + h(Y) = h(Z)$ |
| Unstacking block Y from block Z leaves us with block X | $h(Z) - h(Y) = h(X)$ |
| Blocks X and Y have the same length | $h(X) = h(Y)$ |
| Block X is smaller than block Y | $h(X) < h(Y)$ |
| Stacking n blocks (of type) X adds up to a unit | $h(X)$ estimates $1/n$ |

(H-mappings) and between concrete objects (h-mappings). Table 1 indicates how these alignments between both domains link spatial actions with number operations. Also, Table 2 describes the learning contents addressed by our game: the variables link spatial actions on stackable blocks with the structure of rational numbers mirrored by our game.

The main challenge faced while conceiving the Building Fractions game was to overcome the critique that manipulatives can be detrimental to classroom activities (Ball, 1992), as they can fail to direct attention to the crucial educational features. For example, the intended interpretation of Figure 1 (left) is the addition $2/4 + 3/4$. Without guidance, learners might interpret such visual information as $2 + 3$, $2/5$, or $3/5$. To avoid this problem and promote adequate interpretations, we embedded hints into the game, the crucial one giving cues linking each block with the fraction representing its length. Hence, each block had printed its length on itself. The board game forced players to stack blocks into two specific linear paths having rational numbers printed alongside them (Figure 2). We provided protocols for performing game actions to verbalize the involved fractions. We expected these hints to link each physical block (e.g., from a stacking operation) with the rational number corresponding to its length.

### Theoretical Considerations

In addition to the spatial-numerical analogy, research connecting spatial skills and mathematical learning influenced our design. For instance, we considered research showing that spatial training enhances mathematical abilities (Cheng & Mix, 2014) and experiences of spatial training in classrooms that enhance mathematical knowledge (Lowrie, Logan, & Ramful, 2017).

A vital consideration was the intrinsic integration account (Habgood & Ainsworth, 2011; Habgood, Ainsworth, & Benford, 2005). This account shows that children learn more and play longer when learning contents are embedded into the game's core mechanics. In our game, players must build roads by stacking blocks to develop the game. Comparing these roads (parallel to each other) is crucial for the players to win. Due to the analogical relationship, this comparison of magnitudes becomes a helpful scaffold for comparing fractions. Hence, this comparison of fractions lies at the core of our game's mechanics. By drawing on Kiili et al.'s (2019) classification, we believe that our game addresses the dimensions of magnitude comparison and number line estimation.

On a different note, according to theories of conceptual change, children generate an initial conception of natural numbers that afterward interferes when children try to make sense of rational numbers (DeWolf & Vosniadou, 2015). Comparing 3/4 versus 4/8 may illustrate this "whole number bias" effect as children might ignore the denominators to conclude that 3/4 is lesser than 4/8. However, Figure 1 (right) illustrates how our game prevents such situations by providing spatial information that scaffolds an adequate fraction comparison.

By drawing on these theories, we pursued an intrinsic integration of fractions' conceptual knowledge into our game's mechanics. Hence, we expected that playing the Building Fractions game would help children learn rational numbers. In what follows, we show data from a first exploratory study to assess the educational potential of our game.

## METHODOLOGY

### Sample

We recruited 39 fifth-grade students under high levels of socioeconomic vulnerability (14 men, $M$ = 10.36 years,

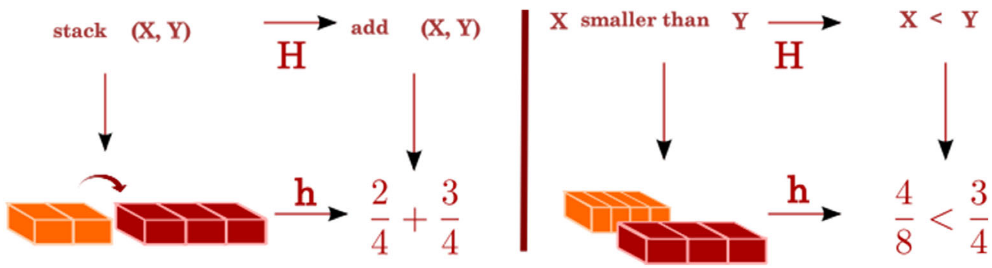


**Fig. 1.** The figure displays the educational analogy that guided the design of the Building Fractions game. On the left, the number of colored blocks represents the numerator of the fractions (2 and 3, respectively), whereas the denominator is represented by the length of each block (1/4). When these two blocks are stacked together, the length of the resulting block corresponds to the addition of 2/4 and 3/4, that is 5/4. On the right, the figure emphasizes blocks that have different lengths. Namely, each red block (1/4) is twice as big as each orange block (1/8). Comparing the lengths of these two blocks supports the inference that 4/8 is a smaller number than 3/4. The H-mappings refer to the relational links between number and space, whereas the h-mappings refer to the concrete links between block lengths and fractional symbols.

*SD* = 0.54) from two schools in a rural area of Chile. Participants and their parents signed informed consent. Each participant was randomly assigned to either the EDG condition ($n = 19$, 5 men) or the entertaining game (ETG) control condition ($n = 20$, 9 men).

### Materials

EDG participants played the Building Fractions game, and ETG participants played a slight variation of the same game. Both games included one board, 26 cardboard stackable blocks with different lengths, a deck of 64 cards, and showed two parallel roads where players could stack their blocks horizontally. Both games looked alike, except that only the EDG had rational numbers imprinted on its elements (see Figure 2). See Appendices S1 and S2 for a detailed description.

### Measures of Knowledge of Fractions and Nonverbal Cognition

To assess conceptual knowledge about fractions, we developed an in-house evaluation designed hand in hand with the learning contents listed in the second column of Table 2. Three expert teachers performed three rounds of reviews to improve this evaluation continually. The final version contained 31 items (Figure 3). These items required participants to translate pictorial representations into fractions (part-whole subconstruct), work with fraction equivalences (ratio subconstruct), and locate values in a number line (measure subconstruct) (Keijzer & Terwel, 2003; Lamon, 2020). The reliability was considered adequate for research purposes (Cronbach's pretest $\alpha = .88$, 95% confidence interval [CI] = [.82, .93], Cronbach's posttest $\alpha = .89$, 95% CI = [.83, .93]).

To measure nonverbal cognitive skills, we applied a fluid intelligence test called TILE. Convergent validity analyses have shown strong correlations between the TILE and academic and mathematical performances (Cerda Etchepare, Pérez Wilson, & Melipillán Araneda, 2019).

### Procedures

We conducted six sessions over 4 weeks. During those weeks, students did not receive any direct instruction about fractions. Throughout the sessions, players were only helped in the game mechanics to keep playing. At the pretest, we assessed participants' conceptual knowledge of fractions (45 min). During each of the four gaming sessions, participants played the games for around 45 min. At the posttest, we re-evaluated participants' knowledge of fractions (45 min), followed by a break (15 min), and applied the TILE test.

### Data Analysis

We computed normalized gains (NGs) (Hake, 1998) to use them in an analysis of covariance (ANCOVA) model where NG was the response variable, Condition was the main predictor, and TILE scores were the covariate. We computed NG's up from the maximum possible score ($M = 31$) and each participant's score at pretest and posttest:

$$\mathrm{NG} = 100 \times \frac{\text{posttest} - \text{pretest}}{M - \text{pretest}}$$

In contrast to traditional analysis, NG scores are easier to interpret. For example, an NG = 20% means that a participant learned 20% of the novel knowledge to which he was exposed (Navarrete 2018; see also Bao, 2006). Previous research indicates that NG scores control learners' initial knowledge as they do not correlate with pretest scores (Hake, 1998; Von Korff et al., 2016).

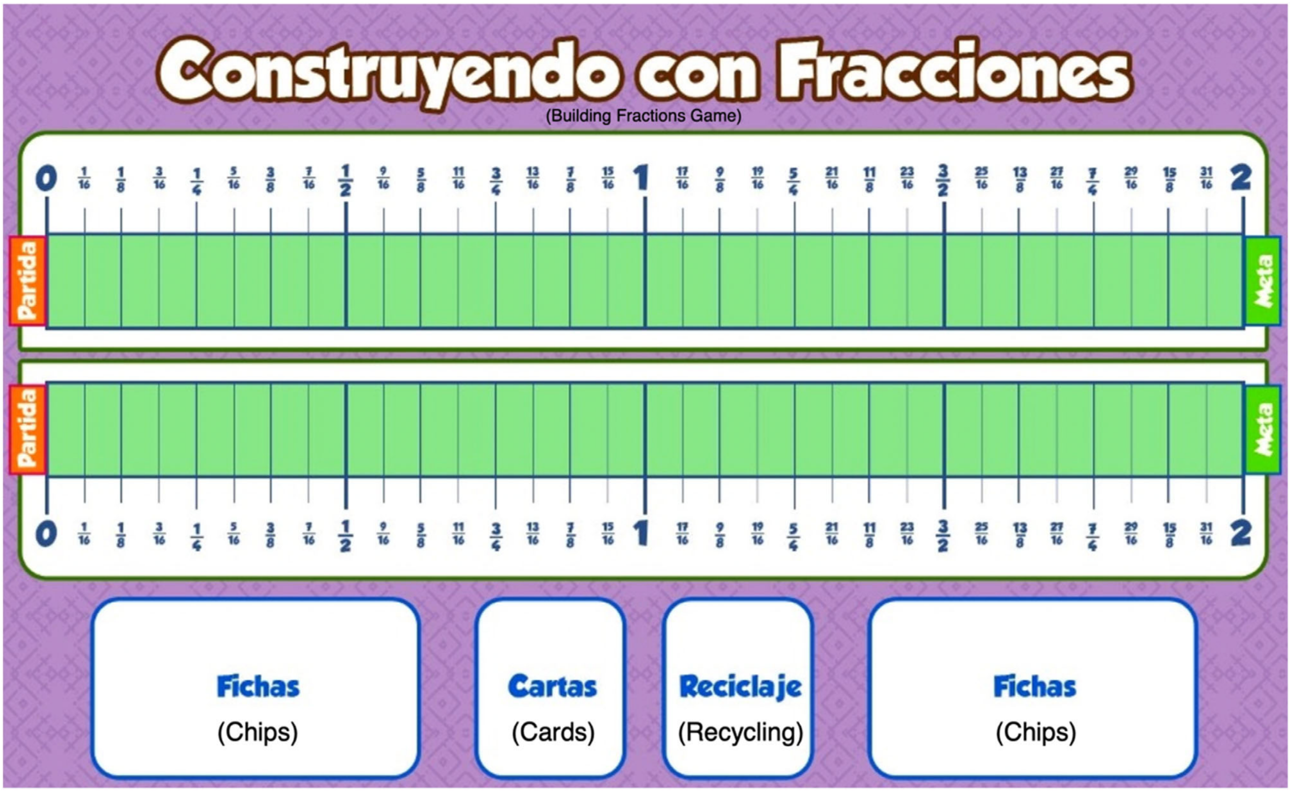


**Fig. 2.** The Board of the Building Fractions game. The board displays two parallel and identical number lines on which blocks have to be stacked by players. A player can understand the value of a fraction as represented by the length of a group of stacked blocks, by reading the fraction annotated in the number line. By having two parallel number lines, participants are also able to compare the sizes of their stacked blocks and their associated fractional values.

### Data Check

Because one EDG participant did not attend the pretest, data analysis considered 18 participants in the experimental condition and 20 participants in the control group. Three participants had to leave before applying the TILE test: we performed a standard imputation method using the MICE package (van Buuren & Groothuis-Oudshoorn, 2011).

## RESULTS

The correlation between NGs and pretest scores was not statistically significant, $r$ = .11, $t(36)$ = 0.63, $p$ = .531, thus suggesting that the baseline knowledge of our participants did not influence the estimated normalized knowledge gains.

The ANCOVA model indicated a main effect of condition, $F(1,35) = 5.25$, $\eta^2_p = .13$, $p = .028$, and a statistically significant effect for TILE scores, $F(1,35) = 5.55$, $\eta^2_p = .14$, $p = .024$, on NGs of knowledge (Figure 4). Both variables were associated with a medium effect size. Figure 4 shows that playing the EDG led players to acquire, on average, 13.26% ($SE$ = 3.96, CI = [5.23, 21.29]) of the novel knowledge of fractions to which they were exposed. In contrast, participants in the entertaining condition showed no benefits (0.73%) from playing the ETG ($SE$ = 3.75, CI = [−6.88, 8.35]).

## DISCUSSION

This report describes the theoretical underpinnings of a board game designed to promote the conceptual knowledge of fractions and provides initial data suggesting its potential to foster the learning of fractions. After controlling for nonverbal abilities and base knowledge, learners who played this game showed more gains than learners playing a similar game lacking the crucial educational analogies. This result supports our hypothesis about the educational benefits of playing this game. Also, from the participants' perspective, they were just playing a game involving some numbers. It was not self-evident that playing a game—without direct teaching guidance—would yield educational gains. Hence, this proof-of-concept suggests that educational analogies can be successfully embedded into board games to scaffold basic and intuitive learning of rational numbers. These results are consistent with the effective use of analogies to

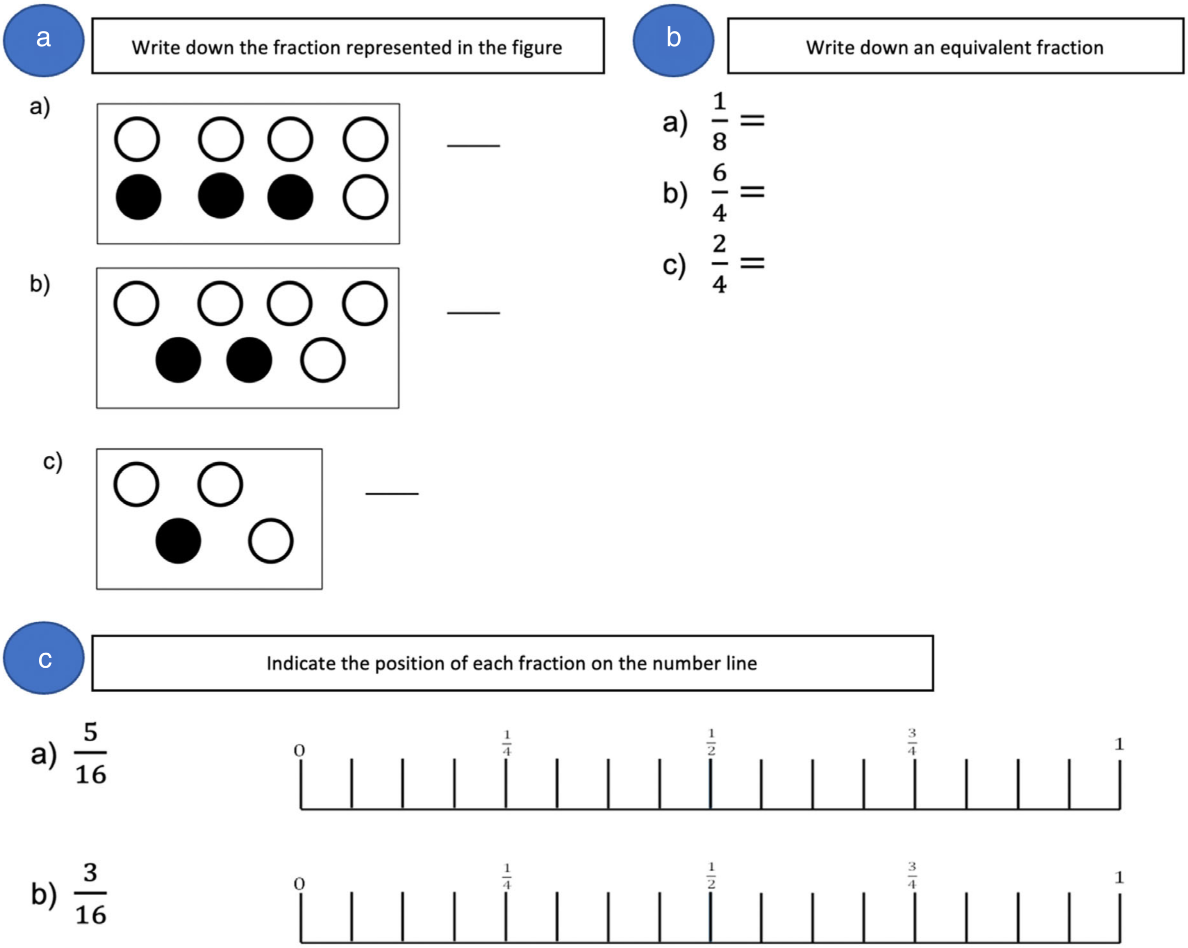


**Fig. 3.** This figure shows three items used in our in-house evaluation. (a) This problem targets the part-whole subconstruct of fractions by asking participants to identify the fraction represented. (b) This problem targets the ratio subconstruct of fractions by asking participants to evoke a fraction equivalent to the cue. (c) This problem targets the measure subconstruct of fractions by asking participants to locate fractions in a number line.

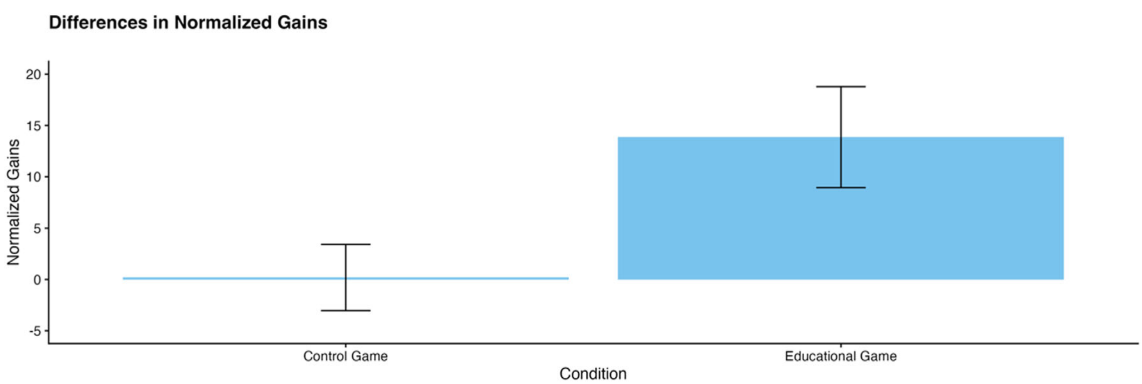


**Fig. 4.** The graph shows the differences between both conditions in terms of normalized gains (NGs). The educational condition showed statistically significant greater gains in fraction knowledge than the control group.

promote learning mathematical subjects such as algebra and arithmetic (Araya et al., 2010; Richland et al., 2007; Richland, Holyoak, & Stigler, 2004; Richland, Stigler, & Holyoak, 2012) and the framework of intrinsic integration (Habgood & Ainsworth, 2011).

Our method does not allow us to argue for the quality of this teaching strategy. Still, our results, at the very least, support the creation of teaching strategies that are indirect, interactive, and exploratory, thus fulfilling the preferences of today's learners (Kirriemuir & McFarlane, 2004). This study has a small sample, so our results should be taken carefully. Also, our participants are under high levels of socioeconomic vulnerability and assist schools located in rural areas, and thus, our results would not generalize to larger populations with dissimilar characteristics. Consider also that the exploratory nature of this study directed our efforts to measure and promote the conceptual knowledge of fractions as a whole, meaning that our data cannot inform specific improvements in particular subconstructs of fractions identified by prior research.

The Building Fractions game outlined here is simple enough for any teacher to use, and thus, we expect it to become a helpful tool for teaching fractions. Due to its minimal cost and ease of implementation, its adoption might be particularly beneficial for learners in schools with high socioeconomic constraints, thus increasing their opportunities to succeed in more advanced math subjects (Booth & Newton, 2012). Hence, this tool might help reduce the gap in understanding fractions that separates less and more affluent children in school.

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**Conflict of interest**

The authors have stated explicitly that there are no conflicts of interest in connection with this article.

## SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

**Appendix S1.** Educational and control games

**Appendix S2.** Supporting Information

**Figure S1**. (A) The graph compares the Educational and Control game conditions by showing differences in residual scores at posttest. This graph embodies the results shown by an ANCOVA model in which posttest scores were used as the response variable. We estimated the residuals from a regression model that included posttest scores as the response variable and pretest and TILE scores as predictors. (B) The graph compares the Educational and Control game conditions by showing differences in residual normalized gains (NGs). This graph represents the results shown by an ANCOVA model in which NGs were used as the response variable. We estimated the residuals from a regression model that included NGs as the response variable and the TILE scores as the only predictor. The similarity between both patterns suggests convergent evidence regarding the difference in gains of knowledge between both groups.

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